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Optimal Harvesting of a Spatial Renewable Resource

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Abstract

In this paper we investigate optimal harvesting of a renewable natural resource. While in the standard approach the resource is located at a single point in space we allow for the resource to be distributed over the plane. Consequently, an agent who exploits the resource has to travel from one location to another. For a fixed planning horizon we investigate the speed and the time path of harvesting chosen by the agent. We show that the agent adjusts the speed of movement so that he accomplishes to visit each location only once, even in the absence of travelling cost. Since he does not come back to any location for a second harvest, it is optimal for him to fully deplete the resource upon arrival. A society interested in conserving some of the resource thus has to take measures suitable to limit the exploitative behaviour of the agent.

JEL-Code: Q200, Q280, D210, C610.

Keywords: optimal harvesting, spatial renewable resource, continuous time, market failure.

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1. Introduction

The economics of optimal harvesting of renewable resources is well established. The fundamental papers for the case of fisheries by Gordon (1954) and Scott (1955) are both more than 50 years old by now. Whereas the former pointed to the problem of overexploitation due to the absence of property rights at sea early, the latter led the path to sophisticated dynamic modelling of optimal resource management that provided the foundation for many more refined research efforts in more recent decades.

Indicating current research trends and opportunities in natural resource economics Deacon et. al. (1998, p. 390) are critical of the plethora of such refinements due to their tendency to suppress important (but technically challenging) details when seeking analytical insights from simpler constructs. As the most important insights from standard models have been obtained, an extension of these models to incorporate some more "real world circumstances" that the managers of fisheries, biologists, and others are concerned with should be attempted.

A most urgent extension of this kind is the recognition of the *spatial dimensions* prevalent in harvesting contexts. Despite its obvious relevance, none of the previous extensions along this line can be found in a recent comprehensive textbooks on the topic (*e. g.* see Conrad, 2010; Perman et al., 2011). Emphasizing this extension, Hannesson (2011a) observes that: "The spatial distribution of fish is rarely analysed in the existing literature, but it could make a difference."

Our aim in this paper is to elaborate on this difference. We thus follow the agenda put forward in Deacon et. al. (1998) who also forcefully demand to increase the realism of resource economics models by acknowledging spatial dimensions: "The spatial dimension of resource use may turn out to be as important as the exhaustively studied temporal dimension in many contexts. Curiously, the profession is only now beginning to move in this direction" (p. 393).

In their survey paper on *The economics of spatial-dynamic processes* Smith, Sanchirico and Wilen (2009) similarly note that whereas there is a long tradition in resource economics to be concerned with the dynamic aspects of resource use, and economics has a long history to address spatial aspects of economics activity, the two approaches have been hardly integrated into a single model ever since Hotelling separated them in his two seminal papers (1929) and (1931).

Recent work that simultaneously allows for spatial characteristics and a time dimension include Sanchirico and Wilen (2005), Costello and Polasky (2008) and the articles cited by Smith, Sanchirico and Wilen (2009). However, due to the technical challenges involved, most of these contributions following the early attempts by Clark (1990) look at (two) distinct and discrete patches of harvesting activity rather than a truly continuous spatial dimension. Similar to Deacon et. al. (1998), Smith, Sanchirico, and Wilen (2009, p.105) conclude that "research addressing integrated spatial-dynamic processes is needed and arguably overdue." Wilen (2007, p. 1135), contrasts this lack of attention by (resource) economists with the prominence of spatial dynamic systems in the "hard sciences" such as mathematics and physics.

While we believe our approach to be more general and broadly applicable, *e. g.* to agricultural and various renewable natural resources, we follow the literature and use the case of fishery for illustration and motivation. The critical point from which virtually all of the existing resource models abstract is that fish is (similar to other resources) distributed spatially, namely in oceans, seas, and rivers. A fisher thus has to travel within the plane by boat to catch fish at each spot he visits.

We assume that the boat starts at some harbour, follows a given route,[1] and eventually returns to its point of departure. The time of this journey (round-trip) depends on the speed of the boat which is controlled by the fisher. We assume that the planning horizon of the fisher is finite. This may be interpreted as either that the fisher is concerned with one fishing (or harvesting) season only, or that the fisher possesses a fishing license with fixed finite maturity, or that the planning horizon equals his working lifetime — and other interpretations may also come to one's mind. For this fixed planning horizon, the number of journeys that can be undertaken equally depends on the speed of the boat. As all fishing is done using fishnets the fisher also has control over the fraction of the stock to be caught by choice of mesh size, boat, and effort.

It is a common precept that society has a concern for sustaining wildlife in the oceans and seas. Also it is well know (as emphasized in Gordon, 1954) that there exist critical externalities resulting from the general absence of property rights. In the case of a renewable resource such as fish, sustainability can also be in the interest of the fisher as a fish stock of a given size that has been harvested will recreate itself after some time allowing for a larger catch in the future. Yet, for any reasonable modelling of costs considerations our model reveals that it will be privately optimal for the fisher to fully exploit the fish stock by only taking the journey once. That is, the fisher goes for one, yet fully exploitative round only, even in the absence of travelling cost. Clearly this outcome cannot be in the interest of society at large. However, as it is very costly or impossible to control

[1]The assumption of a route given at the beginning of the trip does not represent any restriction as long as there is no uncertainty about the location of the resource and hence no necessity to search: We may simply think of the given route as the most lucrative route available, determined beforehand.

the actual catch intensity (see Hannesson, 2011b for detail), society has to consider alternative mechanisms and policies to restrict the amount of fish caught.

One possible remedy for over-exploitation would be to guarantee that the agent undertakes several fishing journeys. With multiple journeys within the fixed time horizon it will then be in his private interest not to over-exploit the resource at his first visit. The underlying idea is that with multiple journeys his tendency to over-exploit the resource is mitigated as the remaining stock will recover after some time providing the potential for an even larger catch in the future. While a minimum number of journeys may be hard to implement with the fisher's effort being unobservable, an equivalent device is to specify the *minimum speed* of the boat that has to be obeyed at all times. Such a policy may be controlled and enforced cost effectively by a continuous threat of punishment (much like the inverse of a speeding ticket).

In order to prevent the fisher from completely exploiting the fish stock immediately and to make him go for several fishing journeys, public policy may, as an alternative instrument, grant the fisher a residual or salvage value for the stock of the resource remaining at the end of the fishing period. Rather than imposing an input control on the fishing activity (*i. e.*, a minimum speed of the boat), this policy directly targets at the fisher's economic incentives: With a salvage payment increasing with the unexploited resource, it is in the fisher's own economic interest not to exhaust the stock unduly. When chosen appropriately, both policy instruments are equivalent in terms of conservation of the stock of the resource.

A minimum speed can force the fisher to arrive at any given location when the stock has not yet reached an attractive size. This makes it worthwhile for him to reduce the catch as, after a first journey, there will remain enough time to harvest the stock again. Rather than letting the stock grow for a longer time and then catching all the fish upon first (and last) arrival, the fisher has an incentive to let the stock grow between journeys as he will return to each given location again. In this way a smaller catch today allows for a larger amount to be caught in the future.

Under this minimum speed policy it becomes privately optimal for the fisher to make sure that a tenable fish stock remains at each location — and thus part of the stock is preserved until the end of the planning horizon. An adequate choice of the minimum speed limit will then guarantee a environmentally attractive and economically sustainable yield, equalizing private and the social objectives. Finally we are able to show that our results and its implications are robust to alternative cost considerations.

The rest of this paper is structured as follows: In Section 2 we describe our formal model. In Section 3 we characterize the agent's optimal harvesting policy, when he is bound to choose a constant speed along with a constant harvesting rate. In Section 4 we demonstrate that our results are robust to possible costs of movement. A detailed analysis of the first period can be found in Section 5. We conclude in Section 6.

2. The Model

Consider an economic agent (we may think of a fisher),[2] exploiting or cutting back a renewable natural resource (*e. g.* a fish stock). Since this resource expands over the plane, harvesting requires the agent to travel from one location to another. That is, when the yield at one particular point in the plane is collected, the agent has to move to the next location in order to proceed with harvesting. Movement, though, may be costly, but this cost can be avoided by reducing the speed of movement — and instead exploiting the resource at each point more severely. While a lower speed implies foregone revenue from the yield of the subsequent points in space that cannot be reached within the given time frame, an intensified extraction leaves the stock with less beneficial conditions for future growth of the resource. The agent thus has to choose both the speed of his movement and the amount to be harvested at each point within this region. We assume that the harvesting capacity of the agent is fixed so that at each point in time the harvest is bounded from above. In other words, we assume that at any point in time and space the agent may collect any nonnegative amount of the resource which neither exceeds the stock nor the fixed harvesting capacity.

For ease of tractability we reduce the dimension of extension of the resource. Instead of literally modelling the extension of the resource and the movement of the agent in a two-dimensional space we consider a one-dimensional setting. We assume that the resource is located on the periphery of a unit circle and that movement of the agent is a travelling on this periphery.[3] We therefore have to keep track of time and location. We denote an instant of time within the fixed time horizon (harvesting period) $\mathcal{T} \equiv [0, T]$ by $t \in \mathcal{T}$; a fixed location by $x \in \mathcal{S} \equiv [0, 2\pi]$; and the location of the agent at time t by $s(t) \in \mathcal{S}$. The size of the stock at location x at time t is denoted by $f(t, x)$. Thus $f(t, \cdot)$ is the distribution of the resource on the periphery at time t.

The natural resource is autonomously growing at rate g. We allow for the growth rate of the stock on a particular location to depend on the stock, but

[2]We henceforth speak of the *agent* and the *fisher* interchangeably.

[3]Clearly, in the real world neither are fishing nets dimensionless nor are fishing grounds one dimensional. However, our model may accommodate such extensions easily, as any two dimensional fishing ground may be projected on a line (here the periphery of a circle), and any one dimensional fishing net on a single point (on that line).

neither on time nor on location directly. That is, we assume that the growth function is constant in time and the same for all locations on the periphery. The growth of the stock is governed by the differential equation $f_t(t,x) \equiv \frac{\partial}{\partial t} f(t,x) = g(f(t,x))$, $\forall x \in \mathcal{S}, t \in \mathcal{T}$,[4] except at some finite set of points of discontinuity of $f(\cdot, x)$. In addition the stock of the resource at location x is reduced by the harvest at time t whenever the agent's location at time t equals x, *i. e.*, $s(t) = x$. If we denote the extraction of the resource at time t at point x by $q(t,x)$ and the agent's harvest by $h(t)$, we have $h(t) \equiv q(t, s(t))$ and $q(t,x) = 0$, $\forall x \neq s(t)$. This reflects the fact that extraction at point x may only take place if x is the actual location of the agent — and if so, the harvest leads to a jump of the stock by $\Delta f(t,x) \equiv f(t^+, x) - f(t^-, x) = -h(t) = -q(t, s(t))$.[5] Thus, for any given location x, the set of arrival times of the agent at x, $J(x) := \{t_1(x), t_2(x), \dots\}$, equals the set of times of (potential) jumps in the stock $f(\cdot, x)$. Putting pieces together, the stock of the resource obeys the law of motion

$$f_t(t,x) = g(f(t,x)) \qquad \forall t \in \mathcal{T} \setminus J(x), x \in \mathcal{S}, \tag{1}$$

$$f(t^-, x) - f(t^+, x) = h(t) \qquad \forall t \in J(x), x \in \mathcal{S}, \tag{2}$$

with $f(0,x) = f_{0x}$ for all $x \in \mathcal{S}$. Note that although h is non-negative for all $t \in \mathcal{T}$, harvesting only leads to a jump in $f(\cdot, x)$ if and when the agent arrives at location x, that is at times $t \in J(x)$.

As the resource is distributed over the periphery of the circle, the agent is required to travel on the periphery in order to proceed with harvesting from one location to the next. There are two natural specifications of how speed may be controlled: Either the agent can control its speed of movement v directly, or it cannot control v directly but acceleration only. We follow the former approach for now. Note that the law of motion of the resource does not depend on the speed of the agent, but on local harvesting activity only.

At any instant of time the agent faces a capacity constraint $\bar{h}$ limiting the possible harvest at each location to $[0, \bar{h}]$. At each moment the agent's harvest is therefore restricted to $h(t) \in H(t) := [0, \min\{\bar{h}, f(t, s(t))\}], \forall t \in \mathcal{T}$. The problem of the agent is then to maximize the discounted profit flow consisting of instantaneous revenue net of instantaneous cost $C(v(t), h(t))$, which generically depends on both speed v and harvest h, for a given planning horizon $\mathcal{T}$. Let $\rho \geq 0$ denote the discount rate of the agent and normalize the price of one unit of the harvested

[4]We assume $g(y) \geq 0$ for at some interval $[\underline{y}, \bar{y}]$.

[5]t^+ denotes the right limit at t: $t^+ \equiv \lim_{s \searrow t} s$; and t^-, the left limit at t: $t^- \equiv \lim_{s \nearrow t} s$.

resource to unity, then his optimisation problem reads as

$$\begin{aligned}
\max_{\{v,h\}} \quad & \int_0^T e^{-\rho t}\left(h(t) - C(v(t), h(t))\right)\, dt \qquad (3)\\
\text{s. t.} \quad & \dot{s}(t) = v(t), \quad \forall t \in \mathcal{T}\\
& f_t(t,x) = g(f(t,x)) \quad \forall t \in \mathcal{T} \setminus J(x), x \in \mathcal{S}\\
& f(t^-,x) - f(t^+,x) = h(t), \quad \forall t \in J(x), x \in \mathcal{S}\\
& h(t) \in H(t), \quad \forall t \in \mathcal{T}\\
& f(0,x) = f_{0x}, \quad \forall x \in \mathcal{S}\\
& s(0) = 0,
\end{aligned}$$

The last condition requires the agent to start, without loss of generality, at location 0. In addition, we may also require the agent to terminate his trip at the point of departure, that is to impose the requirement $s(T) = s(0) = 0$. While this represents a reasonable restriction when the agent can only terminate his trip at home, we allow for the fisher to end the trip at any point (that may also be a harbour) along the route. Yet, as we shall see, even though we allow the agent to go for any real-valued number of rounds, he decides to travel complete rounds only.

3. Constant Speed and Constant Harvesting Rate

From now on we disregard an initial acceleration period and assume for simplicity that the agent may immediately start with some non-negative speed which is maintained until time T. Whence we assume that the agent circulates with some constant speed $v(t) = v$ at all times $t \in \mathcal{T}$.

3.1. Evolution of the stock at a fixed location. Assume that the growth function g is linear,[6] *i. e.*, $g(y) = ry$ and that the initial stock is constant on the periphery of the circle, *i. e.*, $f(0,x) = y_0$. Taken together these conditions imply for each $x \in \mathcal{S} = [0, 2\pi]$:

$$f_t(t,x) = rf(t,x) \quad \text{with} \quad f(0,x) = y_0, \quad \forall t \in \mathcal{T} \setminus J(x), x \in \mathcal{S},$$

yielding

$$f(t,x) = f(0,x)e^{rt} = y_0 e^{rt} =: \tilde{f}(t, y_0),$$

provided that $q(s,x) = 0$, $\forall s \in [0,t]$. Since the agent does harvest, we now derive the evolution of the stock taking into account (interim) harvesting activity.

[6] Clearly, exponential growth cannot prevail for sufficiently large values of the stock due to limited carrying capacities of the environment. So this growth process is to be understood as an

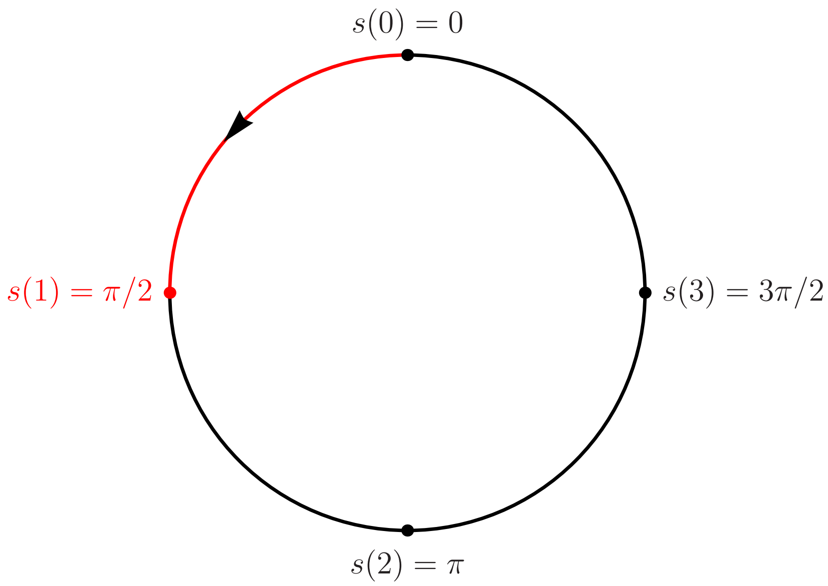

FIGURE 1. Movement of the agent on the periphery with constant speed of $\theta = 4$.

Assuming the agent travels with constant speed along the periphery, *i. e.*, $v(t) = v, \forall t \in \mathcal{T}$, we define the time necessary to pass the periphery of the circle once by $\theta(v) \equiv 2\pi/v$ (see Figure 1). Thus $v/(2\pi) = 1/\theta(v)$ represents the frequency or speed of circling. Simplifying $\theta(v)$ to θ from now on, we may express this and the subsequent formulae in terms of either circling time θ or speed v.

The time of first passage of location $x \in [0, 2\pi]$ then equals $t_1(x) = \frac{\theta x}{2\pi} = \frac{x}{v}$, and the size of the stock at this moment is given by

$$f\left(t_1(x), x\right) = y_0 \exp\left(r\,\frac{\theta x}{2\pi}\right) = y_0 \exp\left(r\,\frac{x}{v}\right).$$

More generally we define $t_n(x)$ as the time of the n-th arrival at location x which is given by

$$t_n(x) \equiv (n-1)\theta + \frac{\theta x}{2\pi}, \qquad \forall n \in \mathbb{N} \setminus \{0\}. \tag{4}$$

Without (interim) harvesting the size of the stock of the resource at the time of the agent's n-th arrival at location x equals

$$y_0 \exp\left((n-1)r\theta + r\frac{\theta x}{2\pi}\right) = y_0 \exp\left(\frac{r}{v}\left(2\pi(n-1) + x\right)\right),$$

which is displayed in Figure 2.

Assuming that at each location x the agent harvests a constant fraction of the stock, say $1-\alpha$ ($\alpha \in [0,1]$), the stock at location x in the n-th period immediately

approximation of the evolution of a stock below a critical size — which constitutes the relevant and interesting case.

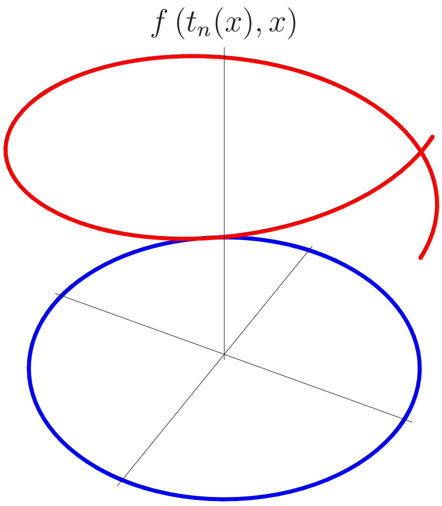

FIGURE 2. Size of the stock of the resource at time of arrival: $f\left(t_n(x), x\right)$.

before the n-th harvest amounts to

$$f\left(t_n(x), x\right) = y_0 \alpha^{n-1} \exp\left((n-1)r\theta + r\frac{\theta x}{2\pi}\right),$$

and accordingly the stock at x *after* the n-th time of passing this location equals

$$y(x, n) \equiv f\left(t_n^+(x), x\right) = y_0 \alpha^n \exp\left((n-1)r\theta + r\frac{\theta x}{2\pi}\right), \qquad \forall n \in \mathbb{N} \setminus \{0\}.$$

Define $y(x, 0) \equiv y_0$. It is important to realize that $y(x, n)$ equals the starting value of the $n+1$ growth period of the resource at location x. With this initial value of each round of growth at hand we can now define the effective growth time since the *last* arrival of the agent at location x:

$$\tau(x, t) = \begin{cases} t & \text{if} \quad 0 \leq t < \frac{\theta x}{2\pi} \\ \operatorname{mod}\left(t - \frac{\theta x}{2\pi}, \theta\right) & \text{if} \quad t \geq \frac{\theta x}{2\pi}, \end{cases}$$

where mod denotes the *modulo*-function. Defining $Q(x, y) \equiv \left\lfloor \frac{x}{y} \right\rfloor$ as the integer quotient of real numbers x and y, $Q(T, \theta)$ denotes the maximal number of complete rounds which can be completed in time T if the time required to complete one round equals θ. Here $\lfloor \cdot \rfloor$ denotes the *floor* function yielding the greatest integer less that or equal to its argument. Correspondingly $\operatorname{mod}(T, \theta) \equiv T - \theta\, Q(T, \theta)$ denotes the time remaining after the maximal number of rounds in time T with circulating time θ has been completed. Consequently the number of times the agent has passed location x at time t is given by

$$m(x, t) = \left\lfloor \frac{t - \frac{\theta x}{2\pi}}{\theta} \right\rfloor + 1.$$

Using y, τ and m we may now derive the resulting stock of the resource at location x and time t:

$$f(x,t) = \tilde{f}\left(\tau(x,t), y(x, m(x,t))\right).$$

The resulting stock is displayed (for $x = \pi/2, \theta = 4, y_0 = 1$ and $\alpha = 2/3$) in Figure 3.

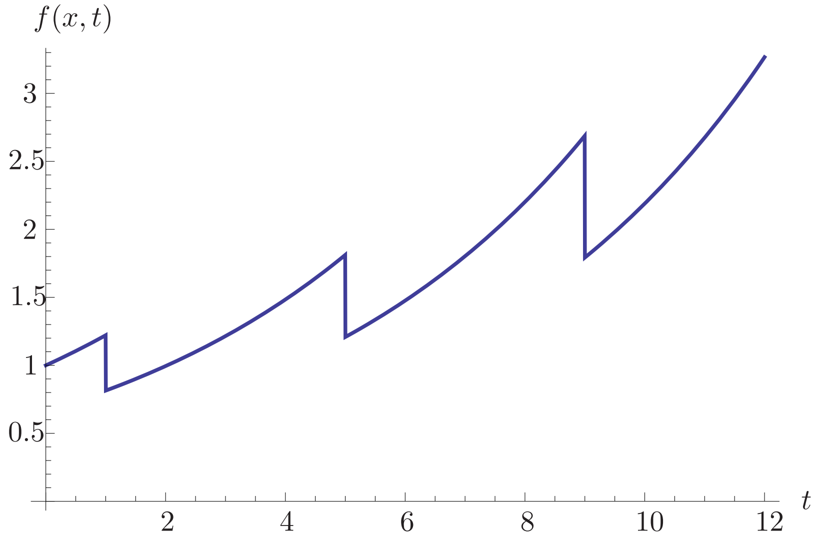

FIGURE 3. Growth of the stock at location $x = \pi/2$ (with $\theta = 4, y_0 = 1, \alpha = 2/3$)

3.2. Aggregate harvest. We now derive the present value of the aggregate harvest. Recall that the discount rate equals $\rho \geq 0$. If we sum up the discounted harvest at location x over all $x \in [0, 2\pi]$ we obtain for the first round of circling

$$\begin{aligned} E(1) &= (1-\alpha)y_0 \int_0^{2\pi} \exp((r-\rho)t_1(x))\, dx \\ &= (1-\alpha)y_0 \int_0^{2\pi} \exp\left((r-\rho)\frac{\theta x}{2\pi}\right) dx \\ &= (1-\alpha)y_0 \frac{2\pi}{(r-\rho)\theta}\left(e^{(r-\rho)\theta} - 1\right), \end{aligned}$$

where we have made use of the fact that $t_1(x) = \frac{\theta x}{2\pi}$ is the time of first passage of location x, and that $\theta = 2\pi/v$ is the time necessary to pass the periphery once.

Similarly the total discounted harvest of the n-th period equals

$$\begin{aligned} E(n) &\equiv (1-\alpha)\alpha^{n-1} y_0 \int_0^{2\pi} \exp(-\rho t_n(x)) \exp\left((n-1)r\theta + r\frac{\theta x}{2\pi}\right) dx \\ &= (1-\alpha)\alpha^{n-1} \frac{2\pi y_0}{(r-\rho)\theta}\left(e^{(r-\rho)\theta} - 1\right) e^{(n-1)(r-\rho)\theta}, \end{aligned} \tag{5}$$

where we have made use of the definition of the general arrival time $t_n(x)$ as defined in (4). More generally, let us define $E(n,x)$ as the discounted harvest of the n-th period *up to location* x, which is given by

$$\begin{aligned} E(n,x) &\equiv (1-\alpha)\alpha^{n-1}y_0 \int_0^x \exp(-\rho t_n(\xi)) \exp\left((n-1)r\theta + r\frac{\theta\xi}{2\pi}\right) d\xi \\ &= (1-\alpha)\alpha^{n-1}\frac{2\pi y_0}{(r-\rho)\theta}\left(e^{(r-\rho)\frac{\theta x}{2\pi}}-1\right)e^{(n-1)(r-\rho)\theta}. \end{aligned} \tag{6}$$

Now $E(x,n)$ is the present value of the harvest of period n if harvesting is only done for locations in $[0,x]$ but not for those in $(x,2\pi]$.

Finally summing the per-period harvest over all periods up to period n we obtain the aggregate harvest after n growth and harvesting periods:[7]

$$\begin{aligned} \sum_{i=1}^{n} E(i) &= (1-\alpha)y_0 \frac{2\pi(\exp((r-\rho)\theta)-1)}{(r-\rho)\theta}\,\frac{(1-\alpha^n\exp(n(r-\rho)\theta))}{1-\alpha\exp((r-\rho)\theta)} \\ &= (1-\alpha)\frac{2\pi y_0}{(r-\rho)\theta}\frac{\left(e^{(r-\rho)\theta}-1\right)\left(\alpha^n e^{n(r-\rho)\theta}-1\right)}{\alpha e^{(r-\rho)\theta}-1} \end{aligned} \tag{7}$$

If $s(T)\neq 0$, that is, if the final period ends before the agent completes the last round-trip we have to add the resulting fraction of the last period. Adding to the above sum the term $E(n+1,s(T))$ we arrive at the discounted aggregate harvest collected within time T as:

$$\begin{aligned} G(\theta,\alpha) \equiv & \sum_{i=1}^{Q(T,\theta)} E(i) + E(Q(T,\theta)+1, s(T)) \\ =&(1-\alpha)\frac{2\pi y_0}{(r-\rho)\theta}\Bigg(\alpha^{Q(T,\theta)}\left(e^{(r-\rho)\mathrm{mod}(T,\theta)}-1\right)e^{\theta(r-\rho)Q(T,\theta)} \\ & + \frac{\left(e^{\theta(r-\rho)}-1\right)\left(\alpha^{Q(T,\theta)}e^{\theta(r-\rho)Q(T,\theta)}-1\right)}{\alpha e^{(r-\rho)\theta}-1}\Bigg). \end{aligned} \tag{8}$$

For any fixed time horizon T, G represents the discounted gross payoff of the agent traveling the periphery with circling frequency $1/\theta$ while harvesting the fraction $1-\alpha$ of the stock at each location. From this gross payoff the agent has to subtract his cost of moving.

We now investigate the properties of G. Whenever the agent completes a full circle, which occurs at location $s(t)=0$, the stock at his present position is discontinuous. This implies that $G(\cdot,\alpha)$ has kinks whenever $\mathrm{mod}(T,\cdot)=0$ and is thus not differentiable at these points. However between any two adjacent kinks,

[7]Note that we do not need to assume $r>\rho$, *i. e.* that the growth rate exceeds the discount rate. In fact, as we show in Appendix A, $E(n,x)$ is always positive and the optimal solution to our problem does not depend on whether $r-\rho$ is positive or negative.

say θ_1 and θ_2, $G(\cdot,\alpha)$ is differentiable and convex as can be seen from Figure 4. Note that $G(\cdot,\alpha)$ is not necessarily quasi-concave, as the green graph in Figure 4 reveals. On the other hand $G(\theta,\cdot)$ is differentiable and quasi-concave as displayed in Figure 5. (In each figure the graph of the lowest parameter value is displayed in red.)

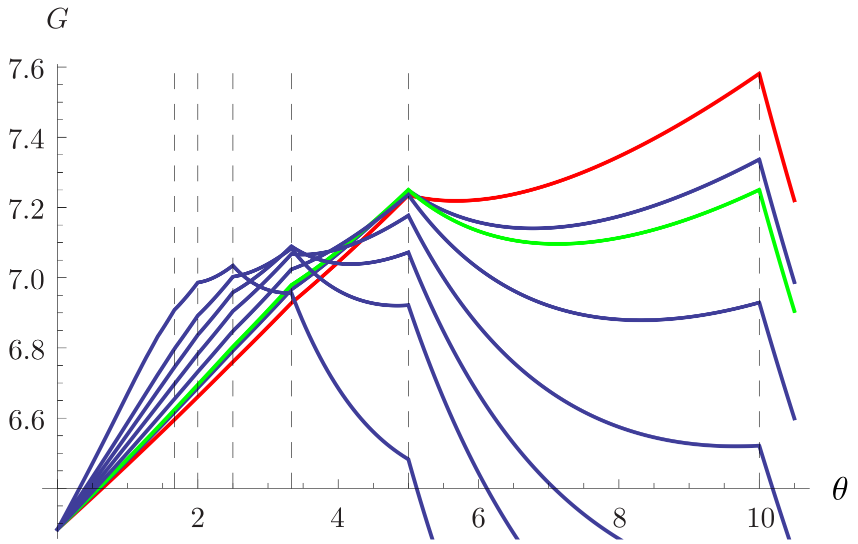

FIGURE 4. G evaluated at $\alpha = 0.07, 0.1, 0.1106$, 0.15, 0.2, 0.25, 0.3, 0.4 (with $y_0 = 1, r = 1/10, \rho = 1/20, T = 10$).

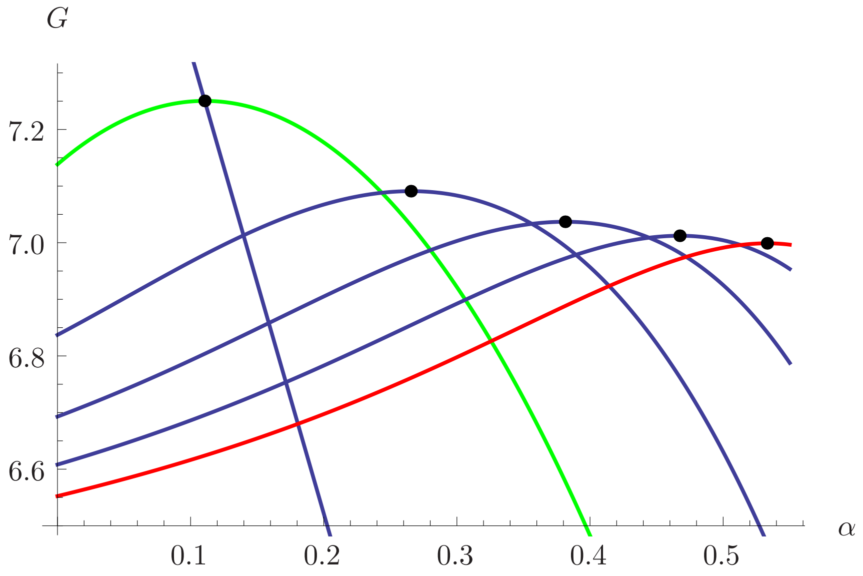

FIGURE 5. G evaluated at $\theta = 10/6, 10/5, 10/4, 10/3, 10/2, 10$ (with $y_0 = 1$, $r = 1/10$, $\rho = 1/20$, $T = 10$).

Quasi-concavity in conjunction with differentiability of $G(\theta, \cdot)$ allows to calculate the optimal harvesting share α^* using the first order condition:

$$\alpha^* : \quad \frac{\partial G(\theta, \alpha)}{\partial \alpha} = 0.$$

As the explicit expression of this derivative provides little insight we relegate it to Appendix B, see equation (B.1). Note, however, that due to lack of quasi-concavity the optimal time for one round cannot be found by differentiation. Yet, as the objective function is convex on any compact interval $[a, b]$ whenever a and b are two adjacent kinks, $G(\cdot, \alpha)$ attains its maximum (if it exists) on either a or b (or both). It thus suffices to compare the values of $G(\cdot, \alpha)$ at a countable number of kinks. Since these kinks accrue whenever the agent adjusts the speed of harvesting so that he manages to complete exactly an integer number of rounds, *i. e.* his final position $s(T)$ equals 2π, we have to compare the discounted payoffs from completing exactly $n = 1, 2, \ldots$ circles within time T. The optimal time of circling θ^* must then equal T/n for some suitable $n \in \mathbb{N}$. In the example used in Figures 4 and 5 we set $T = 10$, so that the kinks appear at $\theta = 10, 5, 10/3, 5/2, 2, 10/6, \ldots$, that is at $n = 1, 2, 3, 4, 5, 6, \ldots$, respectively. These θ-values are indicated by dashed vertical lines in Figure 4.

We now exploit the important finding that in an optimal solution the agent travels an integer numbers of circles $n \in \mathbb{N}$. It follows that $\theta = T/n$ and $\tau = 0$. Observe that with $n = T/\theta \in \mathbb{N}$ the objective function G, defined by equation (8), reduces to the simpler form (7). Similarly we may use this observation to simplify the first order condition for the optimal α. Provided that $e^{\rho\theta} - \alpha e^{r\theta} \neq 0$, condition (B.1) reduces to

$$\begin{aligned} &\alpha^{n-1} e^{nr\theta} \left(\alpha(n(\alpha-1)+1)e^{r\theta} - (n(\alpha-1)+\alpha)e^{\rho\theta}\right) + e^{(n+1)\rho\theta} - e^{\theta(n\rho+r)} = 0 \\ &\Leftrightarrow \quad \alpha^{n-1} e^{n\sigma\theta} \left(\alpha((\alpha-1)n+1)e^{\sigma\theta} - \alpha(n+1)+n\right) - \left(e^{\sigma\theta}-1\right) = 0 \qquad (9) \end{aligned}$$

where we introduced the shorthand $\sigma \equiv r - \rho$. For each given number of rounds of circling n the admissible optimal values of α are given by the positive roots within the unit-interval of the first-order condition (9). Note that the optimal value of α depends on the difference σ only and not on the individual values of r and ρ.

For $n = 1$ equation (9) has no solution suggesting that the optimal value of α equals either zero or unity. As $E(1)$ is linearly decreasing in α, the agent chooses $\alpha(1) = 0$ and harvests the complete stock of the resource. — We investigate this case in detail in the next section.

For $n = 2$ the unique admissible solution of equation (9) is given by

$$\alpha(2) = \frac{1}{2}\left(1 - e^{-\theta\sigma}\right),$$

from which we infer $\frac{\partial \alpha(2)}{\partial \sigma} = \frac{1}{2}\theta e^{-\sigma\theta} > 0$. For $\sigma = 1/20$ and $T = 10$ and thus $\theta = 5$ this formula yields $\alpha(2) = \frac{1}{2}\left(1 - e^{-1/4}\right) \approx 0.1106$, which brings about the maximum of G of the green graph in Figure 5 and the left maximum of G of the green graph in Figure 4. Note that the maximum is not unique, as for this value of α a single round of circling, *i. e.* $\theta = 10$ brings about the same value of G, as $G(5, 0.1106) = G(10, 0.1106) = 7.25046$. However, neither of these constitute a solution of the problem as the total exploitation of the resource yields $G(10, 0) = 8.15207$.

For $n = 3$ we obtain two roots of the first-order condition:

$$\alpha_{1,2}(3) = \frac{1}{3}e^{-\sigma\theta}\left(e^{\sigma\theta} - 1 \pm \sqrt{e^{\sigma\theta} + e^{2\sigma\theta} - 2}\right).$$

Clearly, since $e^{\sigma\theta} + e^{2\sigma\theta} - 2 = (e^{\sigma\theta} - 1)^2 + 3(e^{\sigma\theta} - 1) > (e^{\sigma\theta} - 1)^2$ only the "plus-root" α_1 is positive, and thus represents the unique admissible solution. Differentiating $\alpha_1(3)$ with respect to σ yields

$$\frac{\partial \alpha_1(3)}{\partial \sigma} = \frac{\theta}{\mathcal{N}}\left(4 - e^{\sigma\theta} + 2\sqrt{e^{\sigma\theta} + e^{2\sigma\theta} - 2}\right) > \frac{\theta}{\mathcal{N}}\left(2 + e^{\sigma\theta}\right) > 0,$$

with $\mathcal{N} \equiv 6e^{\sigma\theta}\sqrt{e^{\sigma\theta} + e^{2\sigma\theta} - 2} > 0$.

Finally, for $n = 4$ we obtain three roots of the first-order condition

$$4\alpha^3 e^{3\sigma\theta} + \alpha^2\left(3e^{2\sigma\theta} - 3e^{3\sigma\theta}\right) + \alpha\left(2e^{\sigma\theta} - 2e^{2\sigma\theta}\right) - e^{\sigma\theta} + 1 = 0,$$

and again those within the unit-interval represent the admissible solutions.

The analysis of the cases $n = 2, 3$ and 4 suggests the following results:

- The optimal value of α is increasing with n: The longer the agent chooses to cultivate the resource the lower is the rate of extraction.
- The optimal value of α is increasing with σ: The higher the growth rate of resource (or the lower the agent's discount rate) the lower is the rate of extraction.

The contours of the objective function G are displayed in Figure 6. We have drawn horizontal lines for $\theta = 1, 5/3, 2, 5/2, 10/3, 5$ and 10, *i. e.* for $n = 10, 6, 5, 4, 3, 2$ and 1, respectively. In Figure 6 it is easy to identify the kinks which we have observed in Figure 4. We readily infer from Figure 6 that G has a unique maximum at $(\theta, \alpha) = (10, 0)$ yielding $G(10, 0) = 8.15207$. The agent thus chooses to fully exploit the resource during the single round of circling. A plot corresponding to Figure 6 for $r = 1/5$ is displayed in Figure 7. Comparing the results for $r = 1/10$ and $r = 1/5$ shows that a higher growth rate of the resource does not affect our qualitative results.

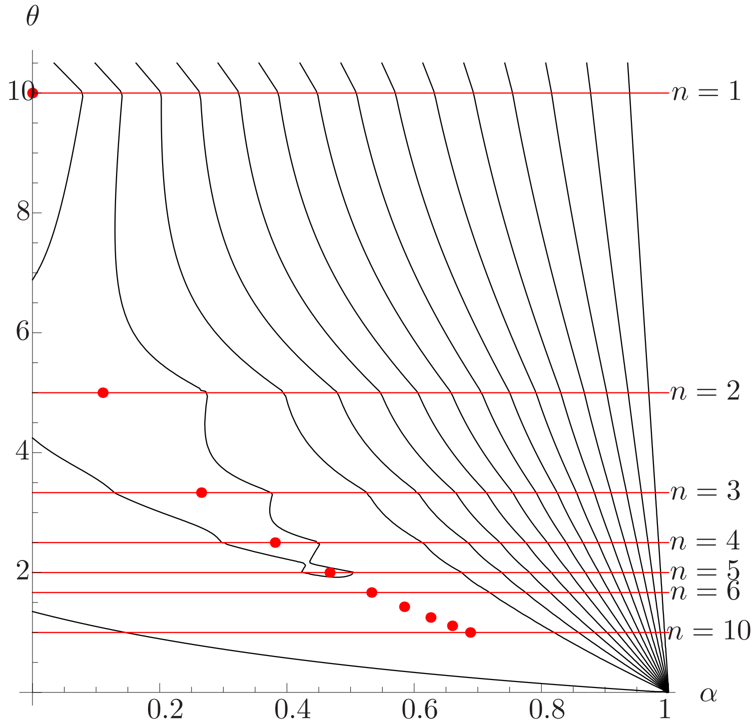

FIGURE 6. Objective function G with $r = 1/10$ ($y_0 = 1, \rho = 1/20, T = 10$).

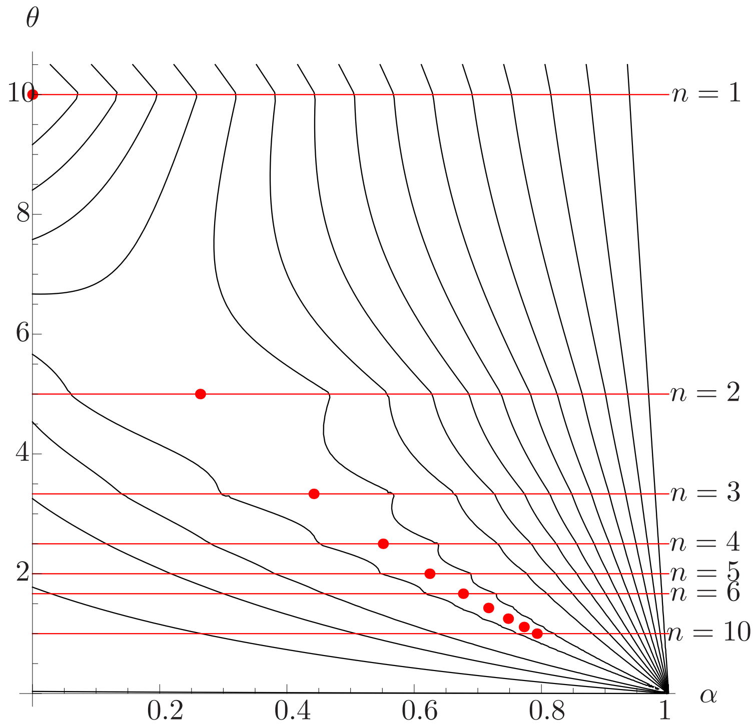

FIGURE 7. Objective function G with $r = 1/5$ ($y_0 = 1, \rho = 1/20, T = 10$).

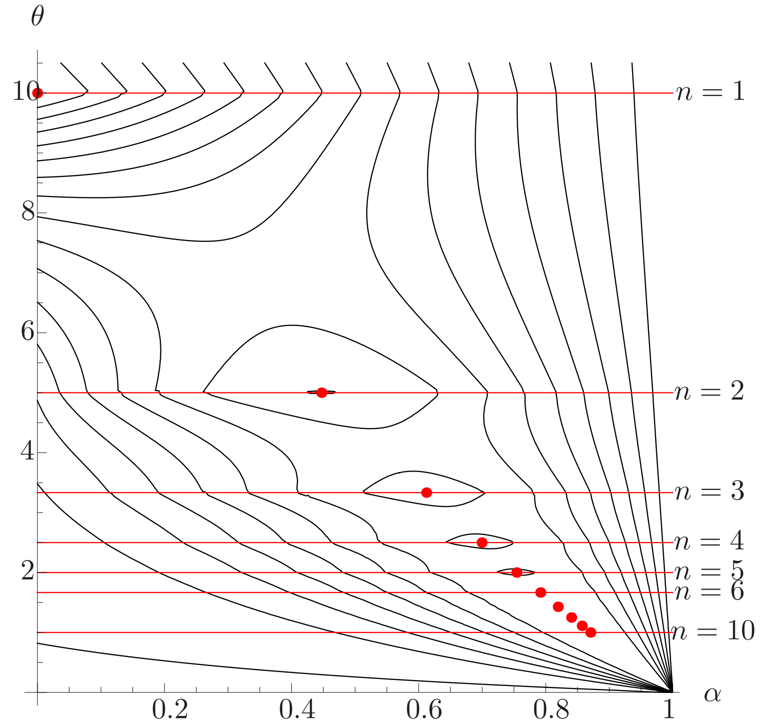

FIGURE 8. Objective function G with $r = 1/2$ ($y_0 = 1, \rho = 1/20, T = 10$).

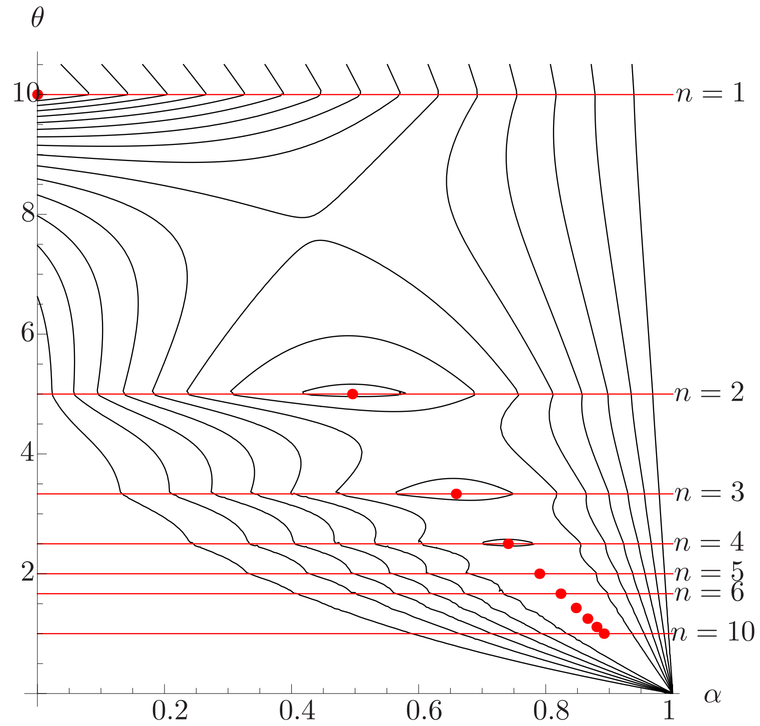

FIGURE 9. Objective function G with $r = 1$ ($y_0 = 1, \rho = 1/20, T = 10$).

It is noteworthy that the structure of the solution of the optimisation problem of the agent within this spatial framework differs from the solution of a corresponding non-spatial model: If, for a resource located at a single point, we maintain the assumptions that the instantaneous profit (payoff) of the agent is linear in harvest and the growth of the resource is linear in both the stock and the harvest, the Hamiltonian is linear in y and h. This linear structure generically leads to "bang-bang" solutions, if the control h is bounded, and to impulse controls, if it is not (with the optimal harvesting policy depending on whether the growth rate r exceeds the discount rate ρ or not).[8]

4. Cost of Movement

Until now we have assumed that the agent may travel on the periphery of the circle at any speed or frequency without incurring a cost. The assumption of a costless choice of speed is innocuous if cost of movement were only dependent on travelling time but independent of speed. However, as travelling cost typically depends on the speed chosen, it is necessary to scrutinize if and how our results change once we account for such costs. To this end let us assume that the agent incurs some constant cost $k > 0$ per unit of distance. With this specification travelling costs increase linearly with speed v. For example the travelling cost for n rounds of circling amounts to $n2\pi k = vT2\pi k$. With this specification the discounted cost of movement of the first round up to location x equals

$$\int_0^x ke^{-\rho t_1(\xi)}\, d\xi = \int_0^x ke^{-\frac{\rho\theta\xi}{2\pi}}\, d\xi = \frac{2\pi k\left(1 - e^{-\frac{\rho x\theta}{2\pi}}\right)}{\rho\theta}.$$

Similarly the discounted cost of movement of the n-th round up to location x amounts to

$$\int_0^x ke^{-\rho t_n(\xi)}\, d\xi = \int_0^x ke^{-\frac{\rho\theta(2\pi(n-1)+\xi)}{2\pi}}\, d\xi = \frac{2\pi ke^{-(n-1)\rho\theta}\left(1 - e^{-\frac{\rho x\theta}{2\pi}}\right)}{\rho\theta}.$$

Summing over $Q(T,\theta) = \lfloor \frac{T}{\theta} \rfloor$ complete rounds plus the residual part $\bmod(T,\theta)$ of the last round we obtain aggregate discounted cost of movement or harvesting as

$$\frac{2\pi k\left(1 - e^{-n\rho\theta}\right)}{\rho\theta} = \frac{2\pi k\left(1 - e^{-\rho T}\right)}{\rho\theta} =: C(\theta).$$

Note that C smoothly depends on the speed of movement θ. Contrary to the aggregate discounted revenue $G(\cdot,\alpha)$ given by equation (8), C does not exhibit kinks and is differentiable everywhere. For that reason the previously observed

[8] For more details on this see, for example, Feichtinger and Hartl (1986), Conrad and Clark (1987), Clark (1990) or Kamien and Schwartz (1992).

kinks of $G(\cdot,\alpha)$ carry over into kinks of the profit function $G(\cdot,\alpha) - C(\cdot)$ for any given value of $\alpha \in (0,1)$.

As C does not depend on α the conditionally optimal harvesting or conservation rate $\alpha(n)$ is not affected by the introduction of a cost of movement. Finally, because $C' < 0$ discounted profit decreases with lower values of θ. As a consequence the introduction of a cost of movement strengthens the agent's incentives to choose a high value of θ. To sum up: the introduction of speed-dependent moving cost fosters our previous result that the agent chooses a complete number of rounds $n = T/\theta$ and that without any additional constraint on n or θ the agent's profit is maximized at choices of $n = 1$ and $\alpha(1) = 0$. The agent visits each location of the resource only once and the stock is completely depleted following this first visit.

Next we wish to explore the consequences of introducing harvest dependent costs. Assume that the speed of circling can freely be chosen at no cost but that harvesting cost amounts to $C(\alpha)$ with $C' < 0$ and $C(1) = 0$. This reflects the idea that a higher harvesting rate increases the cost of harvesting while full conservation of the resource can be accomplished at no cost. As harvesting costs are independent of θ it is easiest to interpret $C(\alpha)$ as per unit of time cost and to integrate harvesting cost over time:

$$\int_0^T c(\alpha)e^{-\rho t}\,dt = \frac{1-e^{-\rho T}}{\rho}\,C(\alpha).$$

The discounted marginal harvesting cost amounts to $\frac{1-e^{-\rho T}}{\rho}c'(\alpha)$ which has to be subtracted from the marginal yield of harvesting derived above (see equation (B.1)). Obviously for marginal cost c' becoming sufficiently large for α approaching zero the complete depletion result obtained above may cease to hold, and interior solutions (*i. e.*, $\alpha > 0$) may result. Moreover a suitable specification of c may even lead to a reversal of the negative relationship between the optimal value of α and θ. As we see no reason for the cost function to exhibit these peculiar features we stop our discussion of what we believe are pathological cases at this point.

5. Detailed analysis of the first period — with a capacity constraint

Assume that the agent chooses to circulate at most once arriving at each point $x \in [0, 2\pi]$ either once or never. We presume here that the agent chooses at most the minimal speed required to travel the full periphery 2π within the given time T. Let $\phi \in [0, 2\pi]$ denote the location of the agent at time T, *i. e.*, $\phi = s(T)$. To be more precise, we should write $\phi(v, T)$ instead of ϕ, since the final position of the agent depends on both the speed he chooses and the travelling time. Using equation (6) the total harvest amounts to

$$E(1,\phi) = \int_0^{\phi} (1-\alpha) y_0 \exp\left(\sigma\theta \frac{x}{2\pi}\right) dx = (1-\alpha)\frac{2\pi y_0}{\sigma\theta}\left(\exp\left(\sigma\theta\frac{\phi}{2\pi}\right) - 1\right).$$

Substituting θ by v we may express total harvest as

$$E(1,\phi) = (1-\alpha)y_0\frac{v}{\sigma}\left(\exp\left(\frac{\sigma\phi}{v}\right) - 1\right) = (1-\alpha)y_0\frac{v}{\sigma}\Big(e^{\sigma T} - 1\Big).$$

The last equality follows from the fact that with constant speed v the agent arrives at location ϕ after travelling time T, *i. e.* $\phi = vT$.

5.1. Non-binding capacity constraint. Suppose that the agent's harvesting capacity $\bar{h}$ is sufficiently high in order for the capacity constraint to be *never* binding within time interval $[0, T]$, *i. e.*, $\bar{h} \geq y_0 e^{r\theta} \geq y_0 e^{rt_1(\phi)}$. Clearly with a non-binding harvesting capacity the agent depletes the total stock at each $x \in [0, \phi]$, for he will never come back to benefit from future growth of the resource. Hence, with $\alpha = 0$ total discounted harvest equals

$$E(1,\phi)\big|_{\alpha=0} \;=\; y_0\frac{v}{\sigma}\left(\exp\left(\frac{\sigma\phi}{v}\right) - 1\right) \;=\; y_0\frac{v}{\sigma}\Big(e^{\sigma T} - 1\Big). \tag{10}$$

If we disregard the cost of movement the problem is nondescript as the agent were to choose the maximally admissible speed, *i. e.*, the minimum speed required to complete a full circle — that is, he were to choose $v^* = \bar{v} \equiv 2\pi/T$, and the final location of the agent at time T is $\phi(v^*) = 2\pi$. Correspondingly, the optimal discounted harvest equals

$$G^* \;=\; E(1, 2\pi)\big|_{\alpha=0} \;=\; E(1)\big|_{\alpha=0} \;=\; y_0\frac{2\pi}{\sigma T}\Big(e^{\sigma T} - 1\Big).$$

However, with the cost of movement being some increasing and strictly convex function of speed $c : [0, \bar{v}] \to \mathbb{R}_+$ the agent has so solve

$$\max_{v} \; y_0\frac{v}{\sigma}\Big(e^{\sigma T} - 1\Big) - c(v), \tag{11}$$

yielding the optimal speed

$$v^* = c'^{-1}\left(\frac{y_0}{\sigma}\left(e^{\sigma T} - 1\right)\right).$$

if and only if

$$c'(0) < \frac{y_0}{\sigma}\left(e^{\sigma T} - 1\right) < c'(\bar{v}).$$

5.2. Binding capacity constraint. In order to broaden the applicability and to enhance the realism of our model, we shall now assume that the agent's harvesting capacity $\bar{h}$ is sufficiently low so that the capacity constraint is *always* binding, *i. e.*, $\bar{h} \leq y_0$, the agent utilizes his full capacity for all $t \in \mathcal{T}$. Hence total discounted harvest equals

$$\int_0^\phi \exp\left(-\rho\frac{\theta x}{2\pi}\right)\bar{h}\,dx \;=\; \frac{2\pi\bar{h}}{\rho\theta}\left(1-e^{-\frac{\rho x\theta}{2\pi}}\right) \;=\; \frac{\bar{h}v}{\rho}\left(1-e^{-\rho T}\right).$$

Disregarding the cost of movement the problem is uninteresting as the agent were to choose $v^* = \bar{v} \equiv 2\pi/T$. With some convex cost function c the agent has to solve

$$\max_v \frac{\bar{h}v}{\rho}\left(1-e^{-\rho T}\right) - c(v), \tag{12}$$

yielding the optimal speed

$$v^* = c'^{-1}\left(\frac{\bar{h}}{\rho}\left(1-e^{-\rho T}\right)\right).$$

5.3. Partially binding capacity constraint. Let us now turn to the more interesting case where the agent's harvesting capacity $\bar{h}$ is intermediate so that $y_0 < \bar{h} < y_0 e^{r\theta}$. In this case the solution is a straightforward combination of the two polar cases above. Define the critical level at which the stock of the resource equals the agent's harvesting capacity $\hat{x}(\bar{h})$ as

$$\hat{x}(h): \quad y_0 \exp\left(r\frac{\theta x}{2\pi}\right) = \bar{h}$$

Thus, $\hat{x}(\bar{h})$, or for short $\hat{x}$, equals

$$\hat{x}(\bar{h}) = \frac{2\pi}{r\theta}\log\left(\frac{\bar{h}}{y_0}\right).$$

Up to time $t_1(\hat{x}(\bar{h}))$ the agent is able to harvest the total stock of the resource. For the remaining time *i. e.* given by the time interval $[t_1(\hat{x}(\bar{h})), T]$ the agent is capacity constrained. The total discounted harvest is then given by

$$\begin{aligned} G(\phi) &= \int_0^{\hat{x}} y_0 \exp\left(\sigma\frac{\theta x}{2\pi}\right)dx + \int_{\hat{x}}^{\phi} \bar{h}\exp\left(-\rho\frac{\theta x}{2\pi}\right)dx \\ &= y_0\frac{v}{\sigma}\left(\frac{r}{\rho}\left(\frac{\bar{h}}{y_0}\right)^{\frac{\sigma}{r}} - \frac{\sigma}{\rho}\frac{\bar{h}}{y_0}e^{-\rho T} - 1\right). \end{aligned} \tag{13}$$

Since G is linear in v the optimal speed is either 0 or $2\pi/T$ depending on whether the term in brackets on the right hand side of equation (13) is negative or positive.

For the special case of *no discounting*, *i. e.* if $\rho = 0$, the analysis proceeds as follows: Without a binding capacity constraint the agent collects a harvest of

$$E(1,\hat{x}) \;=\; \int_0^{\hat{x}} y_0 \exp\left(r\theta\,\frac{x}{2\pi}\right)\,dx \;=\; y_0\frac{v}{r}\left(e^{\frac{r\hat{x}}{v}}-1\right) \;=\; \frac{v}{r}\left(\bar{h}-y_0\right).$$

And for the period with a binding capacity constraint the harvest equals

$$\int_{\hat{x}}^{\phi} \bar{h}\,dx \;=\; (\phi-\hat{x})\,\bar{h} \;=\; (vT-\hat{x})\,\bar{h}$$

With the critical value given by $\hat{x}(\bar{h}) = \frac{2\pi}{r\theta}\log\left(\frac{\bar{h}}{y_0}\right) = \frac{v}{r}\log\left(\frac{\bar{h}}{y_0}\right)$ combining both results yields

$$\frac{v(\bar{h}-y_0)}{r} + \bar{h}v\left(T-\frac{1}{r}\log\left(\frac{\bar{h}}{y_0}\right)\right) \;=\; y_0\frac{v}{r}\left(\frac{\bar{h}}{y_0}(1+rT)-\frac{\bar{h}}{y_0}\log\left(\frac{\bar{h}}{y_0}\right)-1\right),$$

which is linear in v. The agent therefore chooses the maximal speed, *i. e.* $v^* = 2\pi/T$, if and only if

$$w\,(1+rT) - w\log(w) - 1 \;>\; 0,$$

with $w \equiv \frac{\bar{h}}{y_0}$. This in turn implies that the derivative is positive and $v^* = 2\pi/T$, if and only if

$$\frac{\bar{h}}{y_0} \;<\; -\frac{1}{W(-e^{-rT-1})},$$

where W denotes the *Lambert function* or the product logarithm, *i. e.* it gives the principal solution for w in $z = we^w$ (for any complex number z). Since this is a rather mild condition we take it for granted.

6. Conclusion

As put forward in the introduction we believe that our paper makes an important contribution to both the technical side and the policy implications of renewable resource harvesting models that simultaneously look at a time and a spatial dimension. On the technical side we present a simple way to introduce *continuous* spatial dynamics into models of renewable resource harvesting. Countering the startling previous lack of attention by economists given the prominence of spatial dynamic systems in the “hard sciences” as observed by Wilen (2007), we provide a first attempt of a truly dynamic model with continuous variables and solve for the steady state. To this end, we apply the frequently used example of fishery with spatially distributed fish.

On the policy side we are able to show that left to his own devices the agent will choose to go for one, fully exploitative round of harvesting therefore leading to the immediate, *i. e.* at the time of the agent’s first arrival, extinction of the species. An efficient way of preventing this socially harmful result in the absence being able to constantly monitor and control the actual catch intensity is either to require the agent to travel with some minimum speed or to grant his some payment for the stock not depleted at the end of the planning horizon. Both policy measures accomplish to change the agent's action by delaying the speed with which harvesting occurs: With a minimum speed the agent is constrained to undertake more than just one round of harvesting, which compels him to take into account the development of the stock at future times; a payment of the salvage (or terminal) value of the stock provides immediate incentives to account for the resource at the end of the planning horizon. Under both circumstances the agent's private incentives are changed and the social objective of leaving some of the resource to grow in the future may brought in line with his own private interest.

We are convinced that the simplicity of our model and its implications warrant the interest of the economics discipline beyond the realm of resource and environmental economics. The proximity of our model to the two seminal papers of Hotelling (1929, 1931) is at hand. The wealth of applications of his static model (1929) equally applies to our truly dynamic context. Hence, an extension of the present dynamic model to an oligopoly situation is immediate.

Moreover, introducing further realistic properties such as potential movement (or diffusion) of the resource or acceleration of the agent (boat) may enhance the insights from spatial resource models. Similarly, we also expect that recent developments in age-structured capital models (see Hritonenko and Yatsenko, 2006, and Anita, 1998, for references) may fruitfully contribute to a dynamic analysis of the economics of spatially distributed renewable resources. Finally, varying harvesting capacity, that is investment behaviour of the agent, *e. g.* investments of the fisher in boats and fishnets, may constitute a challenging path for future research.

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Appendix A.

We know that total discounted harvest $\sum_{i=1}^{n} E(i)$, given by equation (7), must be neccessarily positive. Since

$$(1-\alpha)\frac{2\pi y_0}{(r-\rho)\theta}\left(e^{(r-\rho)\theta}-1\right) \geq 0 \quad \forall r,\rho \in \mathbb{R}_+,$$

any parameter restriction on $\sigma := r-\rho$ must result from the non-negativity of the expression

$$\psi(n) := \frac{\alpha^n e^{n(r-\rho)\theta}-1}{\alpha e^{(r-\rho)\theta}-1}.$$

Let $\varphi(y) := \alpha^y e^{y(r-\rho)\theta}-1$. It then follows from equation (7) that $\sum_{i=1}^{n} E(i) > 0 \Leftrightarrow \psi(n) \equiv \varphi(n)/\varphi(1) > 0$; and thus $\psi'(n) = \varphi'(n)/\varphi(1)$, with $\varphi'(n) = \alpha^n e^{n\theta(r-\rho)}(\log(\alpha)+\theta(r-\rho))$. Since $\log(\alpha)+\theta(r-\rho) > 0 \Leftrightarrow \varphi(1) = \alpha e^{(r-\rho)\theta}-1 > 0$, we infer that

$$\psi'(n) = \varphi'(n)/\varphi(1) > 0 \quad \forall r,\rho \in \mathbb{R}_+.$$

Actually, we have that $\psi(0) = 0$ and $\psi(1) = 1$. We thus conclude that ψ is positive and strictly increasing for *all* values of r and ρ, although we do not know whether f is decreasing (and negative) or increasing (and positive). Hence, the only parameter restriction on σ amounts to

$$\alpha e^{(r-\rho)\theta}-1 \neq 0 \;\Leftrightarrow\; e^{\rho\theta}-\alpha e^{r\theta} \neq 0. \;\Leftrightarrow\; \sigma \equiv r-\rho \neq \frac{\log\alpha}{\theta}.$$

This condition is given on page 12, immediately before equation (9).

Appendix B.

Differentiating $G(\theta,\alpha)$ with respect to the second argument yields

$$\begin{aligned}
&\frac{2\pi y_0}{\theta\sigma\left(e^{\rho\theta}-\alpha e^{r\theta}\right)^2}\Bigg(-\alpha^n e^{nr\theta-\rho T}\Bigg(-\left(\alpha^2-1\right)e^{2r\theta+\rho\tau}-2\alpha e^{r(\theta+\tau)+\rho\theta}\\
&+2(\alpha-1)e^{r\theta+\rho(\theta+\tau)}+e^{r\tau+2\rho\theta}+\alpha^2 e^{r(2\theta+\tau)}\Bigg)\\
&+n(\alpha-1)\alpha^{n-1}\left(\alpha e^{r\theta}-e^{\rho\theta}\right)e^{n\theta\sigma-\rho\tau}\left((\alpha-1)e^{r\theta+\rho\tau}+e^{r\tau+\rho\theta}-\alpha e^{r(\theta+\tau)}\right)\\
&-2e^{\theta(r+\rho)}+e^{2r\theta}+e^{2\rho\theta}\Bigg)=0. \qquad \text{(B.1)}
\end{aligned}$$

Appendix C.

In order to show that $(n, \alpha) = (1, 0)$ is the maximizer of G, we have to compare $G(i, \alpha(i))$ for all $i = 1, 2, \ldots$. To begin with, we first show that $G(1, \alpha(1)) > G(2, \alpha(2))$. (The values $\alpha(1)$ and $\alpha(2)$ are given on page 12.)

$$
\begin{aligned}
& G(1,\alpha(1)) > G(2,\alpha(2)) \\
\Leftrightarrow & \frac{2\pi y_0 \left(e^{T\sigma}-1\right)}{T\sigma} > \frac{\pi y_0 e^{-\frac{T\sigma}{2}}\left(e^{\frac{T\sigma}{2}}-1\right)\left(e^{\frac{T\sigma}{2}}+1\right)^2}{T\sigma} \\
\Leftrightarrow & \qquad \frac{2e^{\frac{T\sigma}{2}}}{e^{\frac{T\sigma}{2}}+1} > 1.
\end{aligned}
$$

The next step is to show that $G(2, \alpha(2)) > G(3, \alpha(3))$, which is equivalent to

$$
\begin{aligned}
& \frac{\pi y_0 e^{-b/2}\left(e^{b/2}-1\right)\left(e^{b/2}+1\right)^2}{T\sigma} \\
> & \frac{2\pi y_0 e^{-b}\left(e^{b/3}-1\right)}{9T\sigma}\left(-\sqrt{-2e^{2b/3}+e^b+e^{4b/3}}+e^{b/3}+2e^{2b/3}\right) \\
& \times\left(e^{b/3}\left(2\sqrt{-2e^{2b/3}+e^b+e^{4b/3}}+5\right)+2e^{2b/3}+2e^b+\sqrt{-2e^{2b/3}+e^b+e^{4b/3}}\right) \\
\Leftrightarrow & -4e^b\left(\sqrt{-2e^{2b/3}+e^b+e^{4b/3}}+6\right)-9e^{b/6}+7e^{2b/3}+9e^{7b/6}-2e^{4b/3}+5e^{5b/3} \\
& +e^{b/3}\left(12\sqrt{-2e^{2b/3}+e^b+e^{4b/3}}+14\right)-8\sqrt{-2e^{2b/3}+e^b+e^{4b/3}} > 0,
\end{aligned}
$$

where $b \equiv \sigma T$. In a similar manner we may proceed step by step, yet with the terms becoming more and more involved. A numerical analysis, though, readily shows that G is decreasing as the number of rounds increases; more formally, that $\tilde{G}(n) \equiv G(n, \alpha(n))$ is a strictly decreasing function. This is illustrated in Figure 10 for $n = 1, 2, 3$.

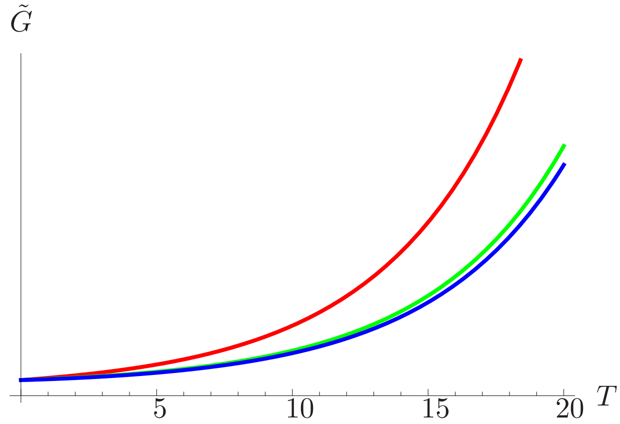

FIGURE 10. Objective function $\tilde{G}(n) \equiv G(n, \alpha(n))$ for $n = 1, 2, 3$ (red, green, blue).