Optimal tax policy under heterogeneous environmental preferences

Marcelo Arbex* **Stefan Behringer**† **Christian Trudeau**‡

May 2, 2017

Abstract

We model an economy of K heterogeneous regions where agents value consumption vs. nature differently. Consumption obtained through pollution-inducing production also generates a negative externality on neighbors. We show that even with a decentralized policy we can obtain first-best efficiency by choosing a combination of pollution taxes in both regions and lump-sum transfers. Moreover, we show that optimal pollution taxes are determined only by the externality parameters, independent of agents' preferences for consumption and nature.

Keywords: Externalities, environmental preferences, optimal taxation.

JEL Classification: D62; H23; H87; Q58.

*,‡ Department of Economics, University of Windsor, 401 Sunset Avenue, Windsor, ON, N9B 3P4, Canada. Email: arbex@uwindsor.ca. † SciencesPo, Department of Economics, 28 rue des Saints-Pères, 75007 Paris, France. Email: stefan.behringer@sciencespo.fr; ‡ *Corresponding Author;* Email: trudeauc@uwindsor.ca;

1 Introduction

We study a K-region economy model where regions face trade-offs between consumption, obtained through pollution-generating economic activities, and the quality of the environment. Pollution not only damages the local environment, but also creates negative externalities on neighbors. We view environmental externalities as generators of public "bads", along the lines of Meade (1952) and his concept of "atmospheric externalities" (Sandmo (2011)). We show that when regions are heterogeneous in three dimensions (nature endowment, damage spillovers and valuation of consumption vs. nature), we can achieve first-best efficiency by using pollution tax rates and a lump-sum transfer together.[1]

Optimal pollution tax rates are determined only by externality parameters. Such Pigouvian taxation aims to charge a region with the social cost of its consumption, and its tax rate is thus increasing in the damage it imposes on its neighbors. The optimal transfer plays a redistributive role and is affected by each region's endowment of nature and the degree of environmental damage spillovers. Wealth is redistributed through lump-sum transfers, irrespective of the economic decisions taken by the regions, while the pollution tax has built-in liability, with the polluter compensating its neighbors.

The inefficiency of decentralized policymaking has long been established as the norm in the theoretical public and environmental economics literature on production efficiency in the face of externalities (Pigou (1920); Samuelson (1954)). In a model with heterogeneous jurisdictions and interjurisdictional environmental damage spillovers, Ogawa and Wildasin (2009) find that decentralized policymaking leads to efficient resource allocation, even in the complete absence of corrective interventions by governments or coordination of policy. Decentralized policymaking can result in globally efficient allocations, even when preferences and production technologies differ among regions and governments care only about local environmental impacts. Fell and Kaffine (2014) argue that Ogawa and Wildasin (2009) result hinges on the fact that in their model, there is a fixed sum of environmental damages across regions, and their central result breaks down if the model endogenizes environmental damages. Even though in our model the sum of environmental damage is affected by the policy choices, the decentralized outcome is still efficient.

2 The Economy

We consider an economy where $\mathcal{K} = \{1, ..., K\}$ regions are inhabited by a large number of agents with identical preferences. We define all variables in per capita terms and consider the case of a constant population. Each region is endowed with an initial environment of quality N_i, which is then reduced by environmental damages (i.e., pollution) e_i linked to production. Each unit of

[1] There is an extensive literature dedicated to alternative policies: carbon taxation, cap-and-trade, tradable permits, and regulations related to pollution control (see, for instance, Montgomery (1972); Baumol and Oates (1988), and Muller and Mendelsohn (2009)).

output produced in region i, labeled as Y_i, results in one unit of environmental damage there. Production in region i has a negative atmospheric externality and causes environmental damage in the other regions. The degree of environmental damage spillovers from the other regions is captured by a region specific parameter $\beta_j \in \left[0, \frac{1}{K-1}\right]$, $\forall j \in \mathcal{K}$, so that the environmental damage experienced by region i is given by

$$e_i = Y_i + \sum_{j \in \mathcal{K} \setminus i} \beta_j Y_j. \tag{1}$$

In our economy, if β_i is strictly positive, local economic activity causes damage not only to the local environment but in other regions as well. Oates and Schwab (1988) assume no interjurisdictional environmental spillovers and environmental quality in any jurisdiction depends only on local economic activity, i.e., $\beta_j = 0$ in equation (1), $\forall j \in \mathcal{K}$, $j \neq i$. The upper limit of $\beta_i = \frac{1}{K-1}$ corresponds to the case in which a unit of output produced in region i does just as much damage elsewhere as it does locally. The analyses of Ogawa and Wildasin (2009) and Fell and Kaffine (2014) are restricted to the case $\beta_i = \beta_j = \beta$, $\forall i, j \in \mathcal{K}$.

The cumulative level of environmental damage is

$$\sum_{i=1}^{K} e_i = \sum_{i=1}^{K} \left[1 + (K-1)\,\beta_i\right] Y_i. \tag{2}$$

We do not assume that the sum of the environmental damage is equal to an exogenous constant. In such a case, the planner can only shift environmental damages across regions, but not reduce aggregate damages. We depart from Ogawa and Wildasin (2009) by allowing the planner to choose (indirectly) the optimal level of environmental damage in each region. Hence, the choice of consumption-nature quality allocations is affected by the heterogeneity of the regions with respect to their preferences for environmental quality and consumption.

The utility function of the representative agent residing in region i is denoted as $u_i(c_i, n_i)$, where c_i is the agent's consumption of a private good in region i and

$$n_i = N_i - Y_i - \sum_{j \in \mathcal{K} \setminus i} \beta_j Y_j \tag{3}$$

denotes nature quality enjoyed locally using equation (1). We make the usual assumptions on utility functions (differentiable, increasing and strictly quasi-concave). We allow for agents in different regions to have different preferences for nature versus consumption.

3 The centralized and decentralized problems

3.1 The centralized problem

Consider an economy where a central government cares equally about agents in all regions and can directly choose production and consumption in all regions, with the constraint that $\sum_{i=1}^{K} c_i \leq \sum_{i=1}^{K} Y_i$.

In order to derive a closed-form expression of the optimal pollution tax, we first write equation (3) in matrix form:

$$\mathbf{BY} \equiv \begin{pmatrix} 1 & \beta_2 & \beta_3 & \dots & \beta_K \\ \beta_1 & 1 & \beta_3 & \dots & \beta_K \\ \beta_1 & \beta_2 & 1 & \dots & \beta_K \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \beta_1 & \beta_2 & \beta_3 & \dots & 1 \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ Y_K \end{pmatrix} = \begin{pmatrix} N_1 - n_1 \\ N_2 - n_2 \\ N_3 - n_3 \\ \vdots \\ N_K - n_K \end{pmatrix} \equiv \mathbf{N} \tag{4}$$

where $\mathbf{B}$ is a (non-singular and non-symmetric) $K \times K$ matrix and $\mathbf{Y}$ and $\mathbf{N}$ are $K \times 1$ vectors. Rearranging equation (4), we have

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ Y_K \end{pmatrix} = \begin{pmatrix} 1 & \beta_2 & \beta_3 & \dots & \beta_K \\ \beta_1 & 1 & \beta_3 & \dots & \beta_K \\ \beta_1 & \beta_2 & 1 & \dots & \beta_K \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \beta_1 & \beta_2 & \beta_3 & \dots & 1 \end{pmatrix}^{-1} \begin{pmatrix} N_1 - n_1 \\ N_2 - n_2 \\ N_3 - n_3 \\ \vdots \\ N_K - n_K \end{pmatrix} \tag{5}$$

Summing up over the Y_i we get

$$\sum_{i=1}^{K} Y_i = \Phi\left(n_1, ..., n_K, N_1, ..., N_K, \beta_1, ..., \beta_K\right)$$

which can be rewritten as

$$\mathbf{1}'\mathbf{Y} = \mathbf{1}'\mathbf{B}^{-1}\mathbf{N}$$

where $\mathbf{1}'$ is the transpose of an $K \times 1$ vector, $\mathbf{B}^{-1}$ is the inverse of the $\mathbf{B}$ matrix and $\Phi = \mathbf{1}'\mathbf{B}^{-1}\mathbf{N}$. We can show (see Appendix for details) that

$$\Phi\left(n_1, ..., n_K, N_1, ..., N_K, \beta_1, ..., \beta_K\right) = \left(\frac{1}{det(\mathbf{B})}\right) \sum_{j \in \mathcal{K}} A_j \left(N_j - n_j\right) \tag{6}$$

where $\Phi(\cdot)$ is a function of exogenous parameters only, and for each $i \in \mathcal{K}$,

$$A_i = \sum_{S \subseteq \mathcal{K} \setminus i} (-1)^{|S|+1} \left(\left(K - |S| - 1 \right) \beta_i + |S| - 1 \right) \prod_{j \in S} \beta_j, \tag{7}$$

$$det(\mathbf{B}) = \sum_{S \subseteq \mathcal{K}} (-1)^{|S|+1} \left(|S| - 1 \right) \prod_{j \in S} \beta_j \tag{8}$$

where S is a coalition of regions.

Re-expressing the problem in terms of consumption and nature levels, c_i and n_i, the planner chooses a first-best allocation by solving the following problem:

$$\max_{\{c_i, n_i\}} \sum_{i=1}^{K} u_i \left(c_i, n_i \right)$$

under the constraint that

$$\sum_{i=1}^{K} c_i = \Phi(\cdot) \tag{9}$$

From the first-order conditions of this problem we obtain the following conditions for first-best efficiency:

$$\frac{\partial u_i(c_i, n_i)}{\partial c_i} = \frac{\partial u_j(c_j, n_j)}{\partial c_j} \tag{10}$$

$$\frac{\partial u_i(c_i, n_i)}{\partial n_i} / \frac{\partial \Phi(\cdot)}{\partial n_i} = \frac{\partial u_j(c_j, n_j)}{\partial n_j} / \frac{\partial \Phi(\cdot)}{\partial n_j} \tag{11}$$

$$\frac{\partial u_i(c_i, n_i)}{\partial c_i} = - \frac{\partial u_j(c_j, n_j)}{\partial n_j} \left(\frac{\partial \Phi(\cdot)}{\partial n_j} \right)^{-1} \tag{12}$$

for all $i, j \in \mathcal{K}$. Condition (10) and (11) impose that the marginal utilities for consumption and nature, respectively, are equalized across regions. According to condition (12), the marginal utility for consumption should be equal to the marginal disutility created by the additional pollution it generates. The feasibility constraint, equation (9), must hold with equality. From this system of equations, we can find the first-best optimal allocations c_i^*, Y_i^*, n_i^*, $\forall i \in \mathcal{K}$.

3.2 The decentralized problem

The timing of the model is as follows. First, the government announces the value of its policy instruments. Second, with this information households in both regions determine their productions, which cause externalities for both regions. Finally, given the decisions of the government and private agents, consumption, nature level and welfare are determined. All parameters $(N_i, \beta_i)_{i \in \mathcal{K}}$ are common knowledge. We solve by backward induction.

For each region i, t_i is the lump-sum transfer households make to (or receive from) the central government and p_i is the pollution tax they pay on each unit of output region i produces. We assume that $\sum_{i=1}^{K} t_i = 0$, i.e., the budget has to balance. We also assume that revenue collected with the pollution tax in region i is transferred to households in other regions, with each of them receiving $\frac{p_i}{K-1}$ per unit of production in region i.[2]

When the central government can make lump-sum transfers and levy a pollution tax, households in region i face the the following budget constraint

$$c_i = Y_i(1-p_i) - t_i + \sum_{j\in\mathcal{K}\setminus i} \frac{p_j Y_j}{K-1}. \tag{13}$$

Hence, region i's household problem is:

$$\max_{Y_i} u_i\left(\left(Y_i(1-p_i) - t_i + \sum_{j\in\mathcal{K}\setminus i} \frac{p_j Y_j}{K-1}\right), \left(N_i - Y_i - \sum_{j\in\mathcal{K}\setminus i} \beta_j Y_j\right)\right). \tag{14}$$

The first-order condition of the region i's household is:

$$\frac{\partial u_i(c_i, n_i)}{\partial c_i}(1-p_i) - \frac{\partial u_i(c_i, n_i)}{\partial n_i} = 0. \tag{15}$$

At the first-best optimal allocations, equation (15) implies:

$$\frac{\partial u_i(c_i^*, n_i^*)/\partial c_i}{\partial u_i(c_i^*, n_i^*)/\partial n_i} = \frac{1}{(1-p_i)}, \tag{16}$$

which combined with equations (11) and (12) implies that the optimal pollution tax in region i is not affected by the households preferences for consumption and nature:

$$p_i^* = 1 + \frac{\partial \Phi(\cdot)}{\partial n_i}. \tag{17}$$

where from equation (6) it follows that $\frac{\partial \Phi(\cdot)}{\partial n_i} = -A_i/det(\mathbf{B})$, which implies that

$$p_i^* = 1 - \frac{A_i}{det(\mathbf{B})}. \tag{18}$$

Interestingly from (7) and (8) as A_i and $det(\mathbf{B})$ depend only on the externality parameters so does the optimal pollution tax p_i^*. It charges region i with the social cost of their consumption. We show that region i's pollution tax is increasing in the degree of environmental damage spillover

[2] For alternative federal, inter-jurisdictional settings see, for instance, Boadway et al. (2013) and Silva and Caplan (1997). The assumption that the central government returns all proceeds of its policies is particularly realistic if we think of supra-national agreements or cases where revenues originating from pollution policies are earmarked to compensate regions that have been polluted.

from region i to other regions, β_i, and decreasing in β_j, the environmental damage experienced by region i due to region j pollution. Although intuitive, these results are not trivial to prove.[3]

Using region i's household budget constraint (13) as well as the optimal pollution tax rate (17), we obtain

$$t_i^* = Y_i^*(1-p_i^*) - c_i^* + \sum_{j\in\mathcal{K}\setminus i} \frac{p_j^* Y_j^*}{K-1}. \tag{19}$$

We summarize our results in the following theorem:

Theorem 1. *Consider an economy of $\mathcal{K} = \{1, ..., K\}$ regions. For any utility functions u_i, initial environment qualities N_j and demand spillovers β_j, $j \in \mathcal{K}$, the planner can obtain the first-best allocation as a result of the decentralized problem by choosing the optimal pollution tax and transfers for $\forall i \in \mathcal{K}$, respectively*

$$\begin{aligned} p_i^* &= 1 + \frac{\partial \Phi(\cdot)}{\partial n_i} \\ t_i^* &= Y_i^*(1-p_i^*) - c_i^* + \sum_{j\in\mathcal{K}\setminus i} \frac{p_j^* Y_j^*}{K-1} \end{aligned}$$

where $\frac{\partial \Phi(\cdot)}{\partial n_i} = -\frac{A_i}{det(\mathbf{B})}$, $\Phi(\cdot)$ is a function of exogenous parameters only and output Y^* and consumption c^* are obtained by solving the centralized problem.

4 Conclusions

We show that even with a decentralized policy we can obtain first-best efficiency by choosing a combination of pollution tax and lump-sum transfers when preferences for consumption and nature are heterogeneous. We show that the optimal pollution tax rates depend only on the externality parameters. The pollution tax is increasing in environmental damage inflicted on other regions and it is decreasing in environmental damage due to others. For arbitrary preferences, optimal transfers depend on regions' preferences and stocks of nature.

Acknowledgments

We thank Roberto Serrano (the editor) for helpful comments and constructive suggestions. We also thank Maria Gallego, Marcelin Joanis, Marcel Oestreich, Emilson Silva, Tracy Snoddon and participants at the 2015 CEA Meetings for helpful comments and discussions, as well as Tselmuun Tserenkhuu for assistance. The third author was supported by the Social Sciences and Humanities Research Council of Canada [grant number 435-2014-0140]. We retain responsibility for any errors.

[3] See the Supplemental Material for comparative statics and additional proofs.

References

BAUMOL, W. J. AND W. OATES (1988): *The Theory of Environmental Policy*, Cambridge University Press.

BOADWAY, R., Z. SONG, AND J.-F. TREMBLAY (2013): "Non-cooperative pollution control in an inter-jurisdictional setting," *Regional Science and Urban Economics*, 43, 783–796.

FELL, H. AND D. KAFFINE (2014): "Can decentralized planning really achieve first-best in the presence of environmental spillovers?" *Journal of Environmental Economics and Management*, 68, 46–53.

MEADE, J. E. (1952): "External Economies and Diseconomies in a Competitive Situation," *The Economic Journal*, 62, 54–67.

MONTGOMERY, W. D. (1972): "Markets in licenses and efficient pollution control programs," *Journal of Economic Theory*, 5, 395–418.

MULLER, N. Z. AND R. MENDELSOHN (2009): "Efficient Pollution Regulation: Getting the Prices Right," *American Economic Review*, 99, 1714–39.

OATES, W. AND R. M. SCHWAB (1988): "Economic competition among jurisdictions: efficiency enhancing or distortion inducing?" *Journal of Public Economics*, 35, 333–354.

OGAWA, H. AND D. E. WILDASIN (2009): "Think Locally, Act Locally: Spillovers, Spillbacks, and Efficient Decentralized Policymaking," *American Economic Review*, 99, 1206–17.

PIGOU, A. (1920): *The Economics of Welfare*, Macmillan and Co., London.

SAMUELSON, P. A. (1954): "The Pure Theory of Public Expenditure," *The Review of Economics and Statistics*, 36, 387–389.

SANDMO, A. (2011): "Atmospheric externalities and environmental taxation," *Energy Economics*, 33, Supplement 1, S4 – S12, supplemental Issue: Fourth Atlantic Workshop in Energy and Environmental Economics.

SILVA, E. C. AND A. J. CAPLAN (1997): "Transboundary Pollution Control in Federal Systems," *Journal of Environmental Economics and Management*, 34, 173 – 186.

Appendix

Closed-form expressions

We first provide two technical lemmas that will be useful to obtain closed-form expressions for Φ and p_i^* and to derive comparative statics results on p_i^*. All proofs are in the Supplemental Material.

Let $\mathcal{K} = \{1, ..., K\}$, $\beta = \{\beta_1, ..., \beta_K\}$ and $\mathbf{B}$ a $K \times K$ matrix such that $\begin{pmatrix} 1 & \beta_2 & \cdots & \beta_K \\ \beta_1 & 1 & \cdots & \beta_K \\ \vdots & \vdots & \ddots & \vdots \\ \beta_1 & \beta_2 & \cdots & 1 \end{pmatrix}$.

Let $M_{ij}(\mathbf{B})$ be the i, j minor matrix of $\mathbf{B}$ (i.e. we remove its i^{th} row and j^{th} column).

Lemma 1. $\det(\mathbf{B}) = \sum_{S \subseteq \mathcal{K}} (-1)^{|S|+1} (|S| - 1) \prod\limits_{j \in S} \beta_j$.

We next show that the determinant is strictly positive.

Lemma 2. *Suppose that* $\mathcal{K} = \{1, ..., K\}$*,* $\beta = \{\beta_1, ..., \beta_K\}$ *and* $\beta_j \in \left[0, \frac{1}{K-1}\right]$ *for all* $j \in \mathcal{K}$*, then* $det(\mathbf{B}) > 0$*.*

Corollary 1. $\det(\mathbf{B}) = \sum_{S \subseteq \mathcal{K}} (-1)^{|S|+1} (|S| - 1) \prod\limits_{j \in S} \beta_j > 0$.

We next find the closed-form expression of $\sum_{j \in \mathcal{K}} (-1)^{j+i} M_{ij}(\mathbf{B})$.

Lemma 3. *For all* $i \in \mathcal{K}$*,*

$$\sum_{j \in \mathcal{K}} (-1)^{j+i} M_{ij}(\mathbf{B}) = \sum_{S \subseteq \mathcal{K} \setminus i} (-1)^{|S|+1} \left((K - |S| - 1) \beta_i + |S| - 1 \right) \prod_{j \in S} \beta_j \equiv A_i.$$

We want to transform $\mathbf{BY} = \mathbf{N}$ into $\mathbf{Y} = \mathbf{B}^{-1}\mathbf{N} = \frac{1}{det(\mathbf{B})} adj(\mathbf{B})\mathbf{N}$. For p_i^* we need to sum the i^{th} column of the adjoint matrix only, which is $\sum_{j \in \mathcal{K}} (-1)^{j+i} M_{ij}(\mathbf{B}) = A_i$.

Theorem 2. $\Phi(n_1, ..., n_K, N_1, ..., N_K, \beta_1, ..., \beta_K) = \left(\frac{1}{det(\mathbf{B})}\right) \sum_{j \in S} A_j (N_j - n_j)$ *and* $p_i^* = 1 - \frac{A_i}{det(\mathbf{B})}$ *for all* $i \in \mathcal{K}$*.*